Does public education hurt the middle class?

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December 11, 2024

Abstract

Using US ACS data and elementary-secondary education finances from 2005 to 2009, I find that middle-class households are more likely than other social classes to opt out of private schools in response to changes in government education spending. To understand the consequences of this finding on income inequality, I construct a model in which individuals decide between private and public education and make transfers to their offspring through investing in their education or giving them a bequest Subsequently, I analyze the effect of three policies on the middle class, namely, an increase in the government spending in public education, an increase in the income tax and an improvement in the productivity of public education. The main finding is that increasing government spending on public education reduces the income of middle-class children by incentivizing parents to opt out of private education.

JEL Codes: E62, I24, D63, E21 .

Keywords: Public and Private education, bequest, human capital, intergenerational transfers.

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1 Introduction

Income inequality is correlated with inter-generational mobility. Countries with the highest income inequality are the least socially mobile. This negative relation between inequality and social mobility is known as the "Great Gatsby Curve" (GGC) (Ermisch et al.,2012). There is substantial research on the drivers that might explain the GGC.[1] In this regard, Jerrim and Macmillan (2015) finds that inter-generational mobility is driven in all countries by educational attainment. In addition, the negative link between income inequality and social mobility is stronger in more unequal countries. Jerrim and Macmillan (2015) provide empirical evidence suggesting that private investment in education, compared to public investment, is larger in more unequal countries. Moreover, these countries spend less on public education and have a higher proportion of children attending private schools or using private tutors. This might be due to the fact that public education has a poorer quality than private education, which is well documented in the literature.[2]

The main goal of this paper is to propose a theoretical framework to explain increased inequality and polarization due to segregation in education and analyze the short-run effect of public education on social classes, focusing on the middle class. This purpose is motivated by two reasons. The first is that a large and strong middle class spurs growth and reduces inequality.[3] The second is the existence of policies the government can take, which has been known to hurt the middle class.[4] Consequently, the government should support and promote policies strengthening the middle class. Our paper provides new insights on how can government intervention in public education be a "curse" on the middle class under specific considerations.

The literature has paid tremendous attention to the role of education in the transmission of (dis)advantage across generations and inter-generational mobility. We contribute to two strands of the literature. The first one examines the role of wealth distribution in explaining inequality through investment in human capital. Galor and Zeira (1993) shows that when investment in education is indivisible, namely characterized by a technological non-convexity, the poorest individuals can acquire education only if they borrow. How-

[1]Check survey on the GGC by Durlauf et al. (2022).

[2]Coleman et al. (1982); Hanushek (1986); Psacharopoulos (1987); Chubb and Moe (1990); Jimenez et al. (1991); Neal (1997); Bedi and Garg (2000); Stevans and Sessions (2000) ; Mizzala et al. (2002); Bettinger (2005); Opdenakker and Van Damme (2006) ;Azimi et al. (2023); Crawfurd et al. (2023).

[3]Easterly (2001) provides empirical evidence that a middle-class consensus determines development outcomes and explains inequality.

[4]Simula and Trannoy (2010) finds that taxation represents a "curse" on the middle class when the government is Rawlsian. As per this paper, when taxes increase, the rich population migrates to other countries with lower tax rates. The middle class, on the other hand, which represents the richer among those who are not rich enough to leave the country, incurs the larger part of the deadweight loss of taxation.

ever, if capital market imperfections create borrowing constraints, low-income individuals are excluded from education, making upward social mobility unattainable. In contrast, rich individuals who inherit a large initial wealth have better access to investment in human capital without the need to borrow. As a result, initial wealth distribution persists and affects the rate of growth and inequality through its impact on the aggregate stock of human capital. Alonso-Carrera et al. (2012) contributes to this line of research by studying the impact of the joint distribution of bequest and human capital as well as fiscal policy on the persistence of inequality in the long run. I differ from this literature by introducing convex human capital technologies. Parents can either invest in public or private education. However, the choice between the type of education is discrete. This allows for the existence of social classes without the disruption of the non-convexity assumption.

The second studies the interaction between inequality and education choice. This literature is identified by the static analysis of education choice (Glomm and Ravikumar, 1998; Hoyt and Lee, 1998; de la Croix and Doepke, 2009, Arcalean and Schiopu, 2015), and the dynamic inequality analysis in a given education regime, either public or private (e.g., Glomm and Ravikumar, 1992; Benabou, 2000; de la Croix and Doepke, 2004). I build on this literature to see particularly how government intervention in an economy with a dual education system has a heterogeneous impact on the education choice of different income groups.[5]

Our paper follows this line of research and contributes to it by introducing a dual education system consisting of public schools financed by the government and private schools, which are assumed to have better productivity than public education. We use an overlapping generations model representing a small open economy in which individuals live for three periods. In the first period, a young individual accumulates human capital by acquiring an education financed by his parent. In the second period, he works, supplies labor, and chooses to educate his offspring in a private or public school. In the third period, he retires. Each parent has one child. As such, we assume no population growth.

In this overlapping generations model, we assume that parents are altruistic and care about the future income of their offspring. Individuals contribute to their children's future income by either giving a physical bequest, investing in education, or both. Consequently, agents derive utility from consumption and the transfers to children. We differ from the literature in three aspects. First, we consider a dual education system in which private schools have better quality in comparison to their public counterparts. Second, we assume a compulsory education system in which the government finances public schools. Finally, we assume that both private and public education technologies are convex.

The model generates four social classes that differ in the quantity in the quantity and

[5]To model a dual education we follow Brotherhood and Delalibera (2019). Other papers using similar education technology include Restuccia and Urrutia (2004).

types of transfers they provide to their children. Depending on the parametric conditions, the economy can exhibit different scenarios. We focus on an economy that is comparable to our empirical data and features three social classes: a poor class that invests in public education and does not provide bequests, a middle class that invests in private education and does not provide bequests, and a rich class that invests in private education and provides bequests. Our findings indicate that depending on specific parameter specifications, government expenditure on education affects the size of these social classes and their subsequent transfers. Specifically, we show that an increase in government spending reduces the middle class and increases the size of the poor class. Moreover, while increased public spending boosts the future income of children from lower-income families attending public schools, it can lower the income of some children from middle-class families by prompting their parents to switch from private to public schools. Improving the productivity of public education has a similar effect when the assumption of private education's superiority is maintained. However, if this assumption is relaxed, inequality decreases without hollowing out the middle class. Regarding taxation, we find that it reduces the optimal investment in education and decreases the future income of rich children, even though bequests increase to compensate for this decrease. It also improves the future income of some middle-class children whose parents choose to opt out of public schools.

The paper is presented as follows. Section 2 gives empirical motivation. Section 3 explains the proposed models and the assumptions upon which the analysis is built. Section 4 shows the equilibrium by solving the problem faced by individuals in this economy. In section 5, we study the effects of governmental intervention in public education on the middle class. Finally, in section 6, we conclude.

2 Empirical motivation

Schettino and Khan (2020) finds that the impoverishment of the middle-income class between the years 2000 and 2014, in reality, started in the 1980s and accelerated as time passed. The main premise of this paper is that government spending on education might be a good reason to explain this phenomenon through its impact on parental decisions over types of schools. We argue that family background and, specifically, educational expenditure is a good predictor of a child's future income. Rich families can send their kids to the most prestigious and the best schools in comparison to poor kids (Skiba et al., 2008). Affluent parents can also invest more in their children's preschool education and tap better early-age educational resources, as well as spending more on after-school training (Fan et al., 2020). Carnevale et al. (2019) assert that in the United States, a kid from a high-income family with low scores in kindergarten has a 70 percent chance of getting a college degree and an entry-level job. In contrast, a kid from a low-income family with high scores in kindergarten has only a 30 percent chance.

What is not clear is the extent to which government spending on education influences middle-class families to choose the best educational option for their kids. In this context, we use US data to evaluate the effect of government spending on the education decisions of middle-class families. We take the IPUMS American Community Surveys (ACS) from the year 2000 to 2019 and Public Elementary–Secondary Education Finance Data from the United states census bureau for the same period.[6] Using this data, we check if the parents' decision over public or private education is elastic to per capita public spending based on their income group. That is, we see the probability of parents opting out of private education when the government spending on education increases. It should be noted that in the theoretical model, we allow all individuals to "privately" invest in the education of their children based on income. This detail is mirrored in the data by the fact that private education is not exclusive to a specific group, although it is most common among the richer social classes. Our empirical model is as follows:

$$Private_i = \beta_0 Inc_i + \beta_1 Exp_i + \beta_2 \sum_{i=1}^{50} S_i + \beta_3 \sum_{j=20}^{65} A_{ji} + \beta_4\ T_t + \epsilon$$

where $Private_i$ is a dummy variable set to take 1 if the household has at least one child in private school, Inc_i is the log of household income, Exp_i represents public expenditure per capita on primary and secondary education for household i, $State_i$ is a state dummy, and Age_{ji} is a dummy representing the age of the household head.

More controls are added to the main specification as a robustness check. These include household education and a vector of racial dummies.[7]

Estimating the regression using a Logit model, we test if the probability of choosing private education for individuals born in families whose income falls between the second and third quartiles of the income distribution reduces when per capita public expenditure in education increases. We control for state dummies to have the net impact of government spending. This is because controlling for the per capita spending alone could capture the effect of this variable on parents' choice through the channel of public education quality.

Results are summarized in Table (1). The estimation results indicate that an increase in per capita education spending is associated with a significant decrease in the likelihood of middle-class families choosing private schooling. In contrast, for poor and rich households, changes in per capita government spending do not show a statistically significant relationship with the probability of enrolling in private schools. We then account for household education and racial composition as robustness checks, and we find virtually

[6] We aggregate the variable to household level a la de la Croix and Doepke (2009).

[7] the racial dummies are for white, Hispanic, Pacific, Black, Asian, and other races.

the same results. Another robustness check is added in the Appendix. It runs the estimation with a metropolitan dummy as a control and uses the total public expenditure instead of primary and secondary public expenditure.

In the remainder of the paper, we build a theoretical model explaining the result we find and the possible consequences this might have on the composition of classes, their optimal decisions, and future inequality.

Table 1: Choice of Private Schooling on public spending per capita

	Middle class	Middle class	Rich	Rich	Poor	Poor
	(1)	(2)	(3)	(4)	(5)	(6)
Pub spending per capita	-0.398***	-0.375***	-0.108	-0.078	-0.208	-0.194
	(-4.780)	(-4.243)	(-0.912)	(-0.693)	(-1.019)	(-0.905)
Household Income	0.697***	0.418***	0.873***	0.796***	0.007	-0.011
	(15.237)	(14.667)	(30.085)	(28.389)	(0.594)	(-1.560)
Household educ	-	0.690***	-	0.638***	-	0.769***
		(24.193)		(15.687)		(20.575)
White	-	0.286***	-	0.237***	-	0.451***
		(6.571)		(6.112)		(8.177)
Hispanic	-	-0.290***	-	0.046	-	-0.449***
		(-3.864)		(0.516)		(-5.679)
Pacific	-	-0.095*	-	0.028	-	-0.486***
		(-1.700)		(0.485)		(-3.238)
Black	-	-0.246***	-	-0.041	-	-0.311***
		(-4.488)		(-0.696)		(-3.719)
Asian	-	0.051	-	0.061	-	0.052
		(1.343)		(1.569)		(0.504)
Other	-	0.220***	-	0.254**	-	0.242**
		(4.259)		(2.513)		(2.250)
Observations	2633037	2633037	1318289	1318289	1287456	1287456
Time fixed effects	✓	✓	✓	✓	✓	✓
State fixed effects	✓	✓	✓	✓	✓	✓

Notes: The dependent variable is a dummy that takes 1 if the household has at least one child in private school. The primary/secondary public expenditure per capita and household income are expressed in logs of constant 2019 dollars. Covariates include household income and race dummies as well as time and age-fixed effects. The poor and rich groups represent the first and last quartiles, respectively. The middle class represents the second and third quartiles. For data sources and summary statistics see Appendix. Standard errors are clustered at the state level and are reported in brackets. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, *** indicates significance at the 1 percent level.

3 The model

We assume a small open economy populated by overlapping generations of individuals who live for three periods. In the first period, a young individual does not consume nor work, he accumulates human capital by attending school, which can be either public or private depending on his parent's decision. In the second period, he works, supplies labor, saves, and chooses between educating his offspring in a public or a private school. In the third period, he retires and allocates his savings between consumption and bequest. Each individual has one child at the beginning of the second period. Hence, there is no population growth. We assume a continuum of adult individuals of constant mass N.[8] Following the convention, we define generation t as the generation whose individuals are adult in period t

Agents derive utility from consumption in the second period t, consumption in the third period $t+1$, and their contribution to the lifetime income of their offspring. Adult individuals contribute to their children's future income by either giving physical bequests, investing in their education, or both. Hence, the utility function of an individual from dynasty i and generation t is as follows:

$$U_t^i = \ln c_t^i + \rho \ln d_{t+1}^i + \beta \ln I_{t+1}^i \tag{1}$$

where ρ and β are strictly positive parameters capturing the temporal discount factor and the intensity of altruism respectively. c_t^i is consumption in the second period, whereas d_{t+1}^i is the consumption in the third period. I_{t+1}^i is the after-tax contribution to the children's lifetime income. A parent contribution to the income of an individual from dynasty i and generation $t+1$ is represented by the following:

$$I_{t+1}^i = (1-\tau) w_{t+1} h_{t+1}^i + b_{t+1}^i \tag{2}$$

where w_{t+1} is the wage per efficiency unit of labor at period $t+1$, $\tau \in [0,1)$ is the tax rate on labor income, h_{t+1}^i is the supplied labor efficiency units from dynasty i and generation $t+1$, and b_{t+1}^i is the bequest given to an individual belonging to dynasty i and generation $t+1$. Since we assume that a parent only has one descendant, the lifetime income of individuals from the same dynasty but different generation could differ if the contribution they receive from their respective parents is different. If the transfer received, on the other hand, is identical, then naturally, the lifetime income would be the same. For individuals belonging to the same generation but different dynasties, the transfers they receive would only be similar if the initial endowments of their respective dynasties were the same. Equation (2) has significant implications for intergenerational mobility, as variations in either the net labor income or the bequest can lead to divergent lifetime incomes among individuals affecting their economic prospects and social mobility.

In this model, we extend the framework established in Alonso-Carrera et al. (2012), introducing a dual education system as an additional variable to capture heterogeneity in educational choices. Specifically, we assume that a child's human capital is determined

[8] As there is no population growth, N is constant throughout

by the parental choice between public and private education, the subsequent investment in their education, and government spending. Let e_t^i denote the investment in children's education of an adult individual of dynasty i and generation t. We also assume that education is compulsory in this economy; that is, all young individuals receive an education. The level of human capital in period $t+1$ for an individual from dynasty i born in period t is determined by the following education technology, as introduced in Brotherhood and Delalibera (2019):

$$h_{t+1}^i = \begin{cases} \alpha(g + e_t^i)^\psi & \text{Public education} \\ (e_t^i)^\psi & \text{Private education} \end{cases} \tag{3}$$

where α is the parameter capturing the quality of public education. We assume that private education has a comparably better quality than public education. Hence, the parameter α is set such that $\alpha \in (0, 1)$. Parents who opt for public education might still have a positive investment in education e (e.g., purchasing books, hiring private tutors, etc.) together with g representing the per capita government's spending on public education. We assume that ψ is strictly less than one. This implies that human capital exhibits decreasing returns to education, which eliminates the possibility of sustained growth from our model.

Parents in this model face a trade-off when deciding between public and private education for their children. Opting for public education allows parents to utilize government funding (g). Still, parents can invest e_t^i to enhance their child's human capital. This investment may include supplementary educational resources, extracurricular activities, or private tutoring to compensate for the lower baseline quality. On the other hand, choosing private education typically requires a higher personal investment (e_t^i) but offers superior educational outcomes, thanks to the inherently higher quality of private institutions. This choice may lead to greater immediate costs but can potentially yield higher human capital (h_{t+1}^i) for the child. This decision is influenced by their current wealth, as wealthier parents may prefer the higher but costlier private education to maximize their child's human capital, while less affluent parents might rely more heavily on public education supplemented with limited or no personal investments.

In this economy, there is a single commodity that could either be consumed or invested, and investment made by adult individuals can either be in physical or in human capital. That is, the income of adult individuals, which is comprised of after-tax wage earnings and inheritance, is distributed between consumption, investment in the child's education, and saving. The budget constraint faced by an adult individual from dynasty i and generation t is then:

$$(1 - \tau)w_t h_t^i + b_t^i = c_t^i + s_t^i + e_t^i \tag{4}$$

with s_t^i, c_t^i, and e_t^i representing the adult individual's savings consumption individual, and the amount he chooses to invest in his child's education. When an individual is in his third and last period, he receives a return on his saving, which is devoted proportionally to his consumption and bequest for his offspring. Therefore, the budget constraint for an individual in the third period is:

$$R_{t+1}s_t^i = d_{t+1}^i + b_{t+1}^i \tag{5}$$

where R_{t+1} is the gross rate of return on saving s_t^i, i.e., $R_{t+1} = 1 + r_{t+1}$. The return on savings is used by the old individual in $t+1$ to be consumed and given as a bequest to his offspring.

By combining (4) and (5), we have the following intertemporal budget constraint:

$$(1-\tau)w_t h_t^i + b_t^i = c_t^i + \frac{d_{t+1}^i + b_{t+1}^i}{R_{t+1}} + e_t^i \tag{6}$$

In this economy, we impose borrowing constraints on parents. By doing so, we avoid that parents borrow to educate their descendants. Becker and Tomes (1986) and Galor and Zeira (1993) have shown that if borrowing is constrained, education introduces intergenerational income persistence when parental investment in education depends only on parental income. When credit markets are perfect, the amount invested in the child's education is independent of parental income. Therefore, a borrowing constraint is necessary to explain intergenerational income persistence. This assumption is captured by the following condition:

$$b_{t+1}^i \geq 0 \tag{7}$$

(5) and (7) implies that $s_t^i \geq 0$; and, hence, borrowing is not possible.

Recall that the model is based on a small open economy. The interest rate, in this framework, is exogenously set in the international capital market such that $r = r^*$, where r^* is the world interest rate. We assume that the good of this economy is produced by means of a production function displaying constant returns to scale on physical and human capital. Moreover, let us assume that the stock of physical capital fully depreciates after one period. As such, the firm's technology can be written as follows:

$$Y_t = F(K_t, H_t) \tag{8}$$

where $H_t = \sum_{i=1}^{N} h_i^t$ is the total supply of efficiency units of labor in period t determined according to the education technology in (3). We can rewrite the production function as follows:

$$y_t = f(k_t) \tag{9}$$

where $y_t = \frac{Y_t}{L_t h_t}$ and $k_t = \frac{K_t}{L_t h_t}$. It should be noted that in this economy, the firms choose the ratio of physical to human capital in a manner consistent with their competitive behavior. That is, the firms' decisions are made such that the marginal productivity equals rental prices of physical and human capital. Based on the assumption of free mobility of physical capital, the ratio of physical to human capital ($\frac{K}{Lh}$) is constant (as $r^* = f'(\frac{K}{Lh})$). Therefore, the wage per efficiency unit of labor in equilibrium is set such that $w = f(\frac{K}{Lh}) - \frac{K}{Lh} f'(\frac{K}{Lh})$. Note that w is constant. Consequently, $w_t^i = wh_t^i$ for all t.

In this model, we assume that the government imposes solely a tax on labour income and spends the revenue to finance public education and on unproductive government spending, G_t^u. Denoting N the total population, the total government spending on public education can be written as the following:

$$G^E = gN\kappa \tag{10}$$

where $\kappa > 1$ is a parameter capturing the government inefficiency or bureaucratic cost. If one unit is devoted to public education by the government, only $1/\kappa$ would be effectively spent to achieve that purpose. g is, as defined in (3), the per capita government's allocation for public education.

The government faces a balanced budget constraint in each period. The total government spending G, is subject to the following condition at period t:

$$G_t = G_t^u + G^E = \int_0^N \tau w h_t^i di \tag{11}$$

In the government budget constraint, we are implicitly assuming that both the per capita spending on education $\bar{g}$, and tax τ, are constant and exogenous.

4 Individuals decisions

In this section, we address the optimization problem faced by an adult individual from dynasty i and generation t, who seeks to maximize their utility as defined in equation (1). The individual makes two distinct types of decisions: decisions on continuous variables and a discrete decision between two education systems. The continuous variables include consumption in the second period (c_t^i), consumption in the third period (d_{t+1}^i), the amount invested in their child's education (e_t^i), and the bequest to their child (b_{t+1}^i); and the discrete decision involves choosing between enrolling their child in public or private education. The optimization is subject to constraints (2), (3), (6), and (7) To effectively solve this problem, we adopt a two-step strategy.

First, for each possible choice of the education system (*public* or *private*) we solve the optimization problem with respect to the continuous variables $\{c_t^i, d_{t+1}^i, e_t^i, b_{t+1}^i\}$. This involves setting up the Lagrangian for each scenario and deriving the first-order conditions to determine the optimal levels of consumption, investment in education, savings, and bequest. During this step, the state variables, namely the inherited bequest b_t^i and the current human capital h_t^i, are treated as given.

Second, after obtaining the optimal solutions for the continuous variables under both public and private education scenarios, we compare the resulting utilities to determine which education system choice maximizes the individual's utility. This comparison allows us to identify whether the individual opts for public or private education based on his total income. Using these indirect utilities, we compute the threshold income at which the individual is indifferent between choosing public or private education. We further discuss the second step later in the paper.

Solving the first step of the problem, we obtain the following optimality conditions (the first-order conditions are computed in Appendix A):

$$c_t^i = \frac{(1-\tau)wh_t^i + b_t^i - \frac{b_{t+1}^i}{R} - e_t^i}{1+\rho} \tag{12}$$

$$d_{t+1}^i = c_t^i \rho R \tag{13}$$

and,

$$\frac{\beta}{I_{t+1}^i} \leq \frac{1+\rho}{R\left((1-\tau)wh_t^i + b_t^i - \frac{b_{t+1}^i}{R} - e_t^i\right)} \tag{14}$$

Equation (12) represents the optimal amount of consumption of an adult individual belonging to dynasty i and generation t, whereas equation (13) characterizes his optimal allocation of consumption along his lifetime. Equation (14) identifies the optimal amount of bequest this individual gives to his direct descendant. and it holds with equality when b_{t+1}^i is non binding, i.e., $b_{t+1}^i > 0$. For the sake of clarity, we can write (14) as the following:

$$\frac{\beta}{I_{t+1}^i} \leq \frac{1}{Rc_t^i}$$

Note that the left-hand side of (14) is the marginal utility gain received by an individual from increasing the amount of bequest b_{t+1}^i given to his child. The right-hand side, on the other hand, represents the marginal utility loss resulting from the decrease in his lifetime consumption because of an increase in the amount of bequest transferred to his offspring. Consequently, condition (14) ensures that when the non-negativity constraint on bequest is non-binding, there is no marginal variation in the utility of parents resulting from giving a larger amount of bequest to their children.

Substituting (12) and (13) in the intertemporal budget constraint in (6), we get the optimal amount of saving s_t^i as a function of h_t^i, b_t^i, e_t^i, and b_{t+1}^i. The latter variables represents the amount of intergenerational transfers. The optimal s_t^i obtained is as follows:

$$s_t^i = \frac{\rho\left((1-\tau)wh_t^i + b_t^i - e_t^i\right) + \frac{b_{t+1}^i}{R}}{1+\rho} \tag{15}$$

When the constraint (7) is non-binding, it is possible to compute the optimal amount of bequest given to the offspring from (14), which is characterized by the following equation:

$$\begin{aligned} b_{t+1}^i &\equiv B(h_t^i, b_t^i, e_t^i) \\ &= \frac{\beta R\left((1-\tau)wh_t^i + b_t^i - e_t^i\right)}{1+\rho+\beta} - \frac{\left((1-\tau)wh_{t+1}^i\right)(1+\rho)}{1+\rho+\beta} \end{aligned} \tag{16}$$

The optimal amount of bequest b_{t+1}^i, as specified in the equation, depends positively on the individual's endowments, h_t^i and b_t^i, and negatively on his investment in the education of his direct descendent, e_t^i.

Regarding the investment in education, parents are faced with two decisions. First, they choose between the two types of schooling systems. Contingent on this choice, children will acquire human capital as defined in (3). Second, adult individuals decide how much to invest in education, depending on the education system initially chosen. Note that the optimal levels of investment in education also vary depending on whether parents choose to make a bequest. All in all, a parent chooses public or private education, then decides how much to invest in his kid's education subject to his decision over bequest. Let us denote $\bar{e}_j$ when the constraint on bequest is non binding, i.e. $b_{t+1}^i > 0$, and $\hat{e}_j$ when the constraint is binding, i.e. $b_{t+1}^i = 0$. Using the first-order conditions in Appendix A, we obtain the following optimal levels of education investment when $b_{t+1}^i > 0$:

$$\tilde{e}_{pu} = \left(\frac{(1-\tau)w\alpha\psi}{R}\right)^{\frac{1}{1-\psi}} - g \tag{17}$$

and,

$$\tilde{e}_{pr} = \left(\frac{(1-\tau)w\psi}{R}\right)^{\frac{1}{1-\psi}} \tag{18}$$

with $\tilde{e}_{pr}$ and $\tilde{e}_{pu}$ representing the optimal level of investment in education in private and public schooling systems, respectively. The optimal investment in both public and private education does not depend on individual factors, including parental income. Moreover, investing in public education is less costly in comparison with private education ($\tilde{e}_{pu} \leq \tilde{e}_{pr}$). Hence, choosing private over public education when (7) is non-binding results in higher human capital for kids.[9] Substituting (17) and (18) in (16), we have two optimal amounts of bequest depending on the type of education parents choose for their direct

[9]this result can be easily proven by substituting (17) and (18) in (3)

decedents, i.e.:

$$
\begin{aligned}
b^i_{pu,t+1} &\equiv B(I^i_t, \tilde{e}_{pu}) \\
&= \frac{\beta R\left(I^i_t - \tilde{e}_{pu}\right)}{1+\rho+\beta} - \frac{(1+\rho)(1-\tau)w\alpha(g+\tilde{e}_{pu})^{\psi}}{1+\rho+\beta}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
b^i_{pr,t+1} &\equiv B(I^i_t, \tilde{e}_{pr}) \\
&= \frac{\beta R\left(I^i_t - \tilde{e}_{pr}\right)}{1+\rho+\beta} - \frac{(1+\rho)((1-\tau)w(\tilde{e}_{pr})^{\psi})}{1+\rho+\beta}
\end{aligned}
\tag{20}
$$

where $b^i_{pu,t+1}$ and $b^i_{pr,t+1}$ are the optimal amounts of bequest when parents invest in public and private education, respectively. Note that (17) and (18) represent the levels of education spending that maximizes the parent's utility the most, thus it is not profitable to educate(invest) more. What if parents can pay more than (17) and (18)? Any extra transfers will take the form of a bequest.

Conversely, when the non-negativity constraint on bequest is binding,i.e. $b^i_{t+1} = 0$, the amount of investment in public and private education is a function of the parent's endowments h^i_t and b^i_t. As shown in Appendix A, if an individual chooses not to leave a bequest and invest in private education, his educational investment would be:

$$
\hat{e}_{pr} = \frac{\beta\psi I^i_t}{1+\rho+\psi\beta} \tag{21}
$$

As for an individual choosing not to give a bequest and investing in public education, we find the following expression for the optimal amount of educational investment:

$$
\hat{e}_{pu} = \frac{\beta\psi I^i_t - (1+\rho)g}{1+\rho+\psi\beta} \tag{22}
$$

Similarly to the public and private investment levels, when the bequest is positive, we immediately obtain that $\hat{e}_{pu} < \hat{e}_{pr}$. The values of investment in education we have in (21) and (22) are the constraint non-optimal levels of education. As an individual's income increases, the constraint relaxes, and education increases. In this case, a parent can invest in education as long as the optimal levels of education $\tilde{e}_{pu}$ or $\tilde{e}_{pu}$ are not reached. Once $\hat{e}_{pu} = \tilde{e}_{pu}$ or $\hat{e}_{pr} = \tilde{e}_{pu}$, parents start giving bequest to their descendants.

Additionally, we impose a non-negativity constraint on both public education investment in (17) and (22) such that:

$$
\hat{e}_{pu} \geq 0 \quad \text{and} \quad \tilde{e}_{pu} \geq 0 \tag{23}
$$

This condition ensures that we do not have a negative investment in public education as government spending per capita increases. We discuss the implications of (23) later in the paper.

Based on the previous results, I summarize four distinct outcomes determined by the type of schooling chosen, the subsequent level of investment, and whether or not a bequest is provided to direct descendants. From here onward, we will categorize each choice as belonging to a specific social class. These social classes are distinguished as follows:

- *Social class 1:* does not give bequest and invests $\hat{e}_{pu}$ in public education.
- *Social class 2:* gives bequest $b^i_{pu,t+1}$ and invests $\tilde{e}_{pu}$ in public education.
- *Social class 3:* does not give bequest and invests $\hat{e}_{pr}$ in private education.
- *Social class 4:* gives bequest $b^i_{pr,t+1}$ and invests $\tilde{e}_{pr}$ in private education.

We move to the second step of the individual's optimization problem. In this step, the individual must make a discrete decision between two education systems: public and private, and decide whether to leave a bequest. Formally, the parent chooses one of the four social classes. For this, the decision hinges on which system yields the higher utility for the individual based on their total income and the utility outcomes from the previous step. Specifically, the individual compares the utility values obtained under each social class, using the derived optimal consumption and investment levels, to determine which class maximizes his overall lifetime utility. The decision rule is straightforward: the individual will choose a social class if the utility from this decision exceeds that of choosing the remaining classes.

To facilitate the comparison of utility outcomes across different social classes, we introduce the indirect utility functions for each social class. We denote these functions as V_{i} such that $i = \{1, 2, 3, 4\}$ representing social Classes 1 through 4, respectively. The indirect utilities represent the maximum attainable utility an individual can achieve under each specific set of choices regarding consumption, education investment, and bequests. By substituting the optimal values of the continuous variables derived in the first step into the individual's utility function (1), each indirect utility encapsulates the lifetime utility associated with a particular social class. This allows individuals to evaluate and compare the overall utility derived from selecting any of the four social classes given their economic endowments. The indirect utilities are:

$$V_1(I_t, \hat{e}_{pu}) \equiv (1+\rho+\psi\beta)\ln\left(\frac{I^i_t + g}{1+\rho+\psi\beta}\right) + \beta(1-\psi)\ln\left(\frac{\tilde{e}_{pu}+g}{\beta\psi}\right) + \rho\ln\rho R + \beta\ln\beta R \tag{24}$$

$$V_2(I_t, \tilde{e}_{pu}) \equiv (1+\rho+\beta)\ln\left(\frac{I^i_t - \tilde{e}_{pu} + \frac{g+\tilde{e}_{pu}}{\psi}}{1+\rho+\beta}\right) + \rho\ln\rho R + \beta\ln\beta R \tag{25}$$

$$V_3(I_t, \hat{e}_{pr}) \equiv (1+\rho+\psi\beta)\ln\left(\frac{I^i_t}{1+\rho+\psi\beta}\right) + \beta(1-\psi)\ln\left(\frac{\tilde{e}_{pr}}{\beta\psi}\right) + \rho\ln\rho R + \beta\ln\beta R \tag{26}$$

$$V_4(I_t, \tilde{e}_{pr}) \equiv (1+\rho+\beta)\ln\left(\frac{I^i_t + \frac{1-\psi}{\psi}\tilde{e}_{pr}}{1+\rho+\beta}\right) + \rho\ln\rho R + \beta\ln\beta R \tag{27}$$

Before comparing the indirect utilities across different social classes, we highlight two fundamental points that affect our analysis. First, in certain cases, choosing a particular social class might always be optimal regardless of the parent endowments. For example, if the indirect utility V_2 is strictly greater than V_1 for every I_t, then parents would always prefer to belong to social class 2. Second, some social classes are only feasible when specific conditions are met. This is essential to maintain the integrity of the model. Specifically, we consider four conditions that are a logical consequence of the non-negativity constraints on bequest (7) and public education investment (23). Two are an implication of the non-negativity constraint on bequests. and the remaining two are an implication of the public investment constraint.

Starting with the non-negativity constraint on bequests, recall that individuals in social classes 2 and 4 are characterized by making positive bequests to their descendants, satisfying the non-negativity constraint on bequests given by equation (7). That is, $b^i_{t+1,pu} > 0$ and $b^i_{t+1,pr} > 0$. This requirement implies the existence of income thresholds above which these positive bequests are possible, making social classes 2 and 4 feasible options. Using (19) and (20), we obtain I_2 which is the level of income at which $b^i_{t+1,pu} = 0$ and I_4 the level of income when $b^i_{t+1,pr} = 0$. These thresholds are calculated as follows:

$$I_2 = \frac{(1+\rho+\psi\beta)\tilde{e}_{pu} + (1+\rho)g}{\psi\beta} \tag{28}$$

and,

$$I_4 = \frac{(1+\rho+\psi\beta)\tilde{e}_{pr}}{\psi\beta}. \tag{29}$$

From (28) and (29), we can draw two key conclusions. First, I_4 is greater than I_2.[10] Second, individuals with an income strictly higher than I_4 (I_2), can belong to social class 4 (2) because they can afford to make positive bequest while investing the optimal amount in private (public) education. It is important to note that exceeding these income thresholds does not guarantee that parents will choose these social classes; rather, it makes these classes feasible options. In other words, the non-negativity constraint on bequest (7) limits the availability of certain social classes based on the individual's income. To illustrate, generic individuals with income $I^i_t < I_4$ can not belong to social class 2 or 4 because they are not willing to leave positive bequests while investing the optimal level in public education. Their feasible options are limited to social classes 1 and 3. Similarly, generic individuals with an income satisfying $I_2 \leq I^i_t < I_4$ can not belong to social class 4 for the same reasons. Their feasible options are social classes 1,2 and 3. As for individuals with income $I^i_t > I_4$, all four classes become feasible options. The intuition is as follows. Rich parents can choose any education system as long as it maximizes their respective

[10] we rearrange (28) as as $I_2 = \frac{(1+\rho+\psi\beta)(\tilde{e}_{pu}+g)}{\psi\beta} - g$. We know from (17) and (18) that $\tilde{e}_{pu} + g < \tilde{e}_{pu}$, therefore, $I_2 < I_4$

utility, while poorer parents have limited choices.

Second, in addition to the borrowing constraint discussed earlier, there are two other conditions arising from the non-negativity constraint of public education investment. Specifically, Equations (17) and (22) must be positive (Condition 23). We analyze each implication of this constraint separately.

The inequality $\tilde{e}_{pu} \geq 0$ implies:

$$g \leq \bar{g} \quad \text{where} \quad \bar{g} = \left(\frac{(1-\tau)w\alpha\psi}{R} \right)^{\frac{1}{1-\psi}}. \tag{30}$$

This parametric condition sets a threshold for government spending that must not be exceeded for parents to invest a positive amount in public education. The intuition here is clear. If g is high enough, parents will not find it optimal to invest extra into the human capital of their direct descendants as a substitution effect exists between government spending and private investment in public education. This setup creates a conditional response based on the exogenously set value of g where the optimal investment amount is zero once government spending surpasses the threshold. We represent this outcome as follows:

$$\tilde{e}_{pu} = \begin{cases} \left(\frac{(1-\tau)w\alpha\psi}{R} \right)^{\frac{1}{1-\psi}} - g & \text{if} \quad g < \bar{g} \\ 0 & \text{if} \quad g \geq \bar{g} \end{cases} \tag{31}$$

The inequality $\hat{e}_{pu} \geq 0$ implies the existence of a threshold income level, denoted I_0, above which $\hat{e}_{pu}$ is guaranteed to be positive. By rearranging this inequality, we derive the threshold income I_0 as follows:

$$I_0 = \frac{(1+\rho)g}{\beta\psi}. \tag{32}$$

It is important to note that this threshold I_0 is lower than I_2. In fact, I_2 equals I_0 only when $\tilde{e}_{pu} = 0$. When these two thresholds are equal, we end up with an economy where parents choosing public education do not invest additional resources, as government spending alone satisfies the educational investment needs. This applies to both social classes 1 and 2.

To obtain the optimal class for each individual, we compute the thresholds at which individuals are indifferent between belonging to one social class or another. We get $V_4 > V_2$ if and only if the following condition is fulfilled:

$$(1-\tau)w\tilde{e}_{pr}^{\psi} - R\tilde{e}_{pr} > (1-\tau)w\alpha(\bar{g} + \tilde{e}_{pu})\psi - R\tilde{e}_{pu} \tag{33}$$

This equation represents the condition under which a parent will always choose to belong to social Class 4 (private education with bequests) over social Class 2 (public education with bequests). Specifically, the net return from investing an amount $\tilde{e}_{pr}$ in private education which is calculated as the offspring's future after-tax wage income from private education $(1-\tau)w\tilde{e}_{pr}^{\psi}$ minus the opportunity cost of this investment $R\tilde{e}_{pr}$, must exceed the net return from public education $((1-\tau)w\alpha(\bar{g}+\tilde{e}_{pu})^{\psi}-R\tilde{e}_{pu})$. By assuming that Condition (33) holds throughout the analysis, the model focuses on scenarios where higher-income individuals prioritize private education to maximize their child's future income, thereby reinforcing intergenerational income persistence.

we further have that $V_2 > V_1$ and $V_4 > V_3$ for specific income levels. These relationships are formally presented in the following proposition:

Proposition 1. *If $I_t > I_2$, then $V_2 > V_1$, and if $I_t > I_4$ then $V_4 > V_3$.*[11]

Summarizing, we establish that $V_4 > V_2 > V_1$ and $V_4 > V_3$, with I_2 and I_4 defining social classes 2 and 4, respectively. Additionally, the threshold I_0 identifies the subgroup of individuals who choose social class 1 but do not invest in public education. However, V_2 and V_1 are not comparable with V_3, necessitating the identification of additional threshold conditions to establish a ranking among them. This implies the existence of two additional thresholds. The first identifies the generic individual indifferent between belonging to social classes 1 and 3. The second identifies the generic individual indifferent between belonging to social classes 2 and 3. To find these thresholds, we solve the following equations:

$$V_1(I_t) = V_3(I_t) \tag{34}$$

$$V_2(I_t) = V_3(I_t) \tag{35}$$

Let's denote the solutions for (34) and (35) to be I_1 and I_3 respectively. Using the indirect utilities previously defined, we get that the solution for (34) is:

$$I_1 = \left(\frac{\alpha^{\frac{\beta}{1+\rho+\psi\beta}}}{1-\alpha^{\frac{\beta}{1+\rho+\psi\beta}}} \right) g \tag{36}$$

As for (35), there is no explicit solution. However, we can obtain insights by using an auxiliary function that we define as the difference between V_3 and V_2. Let us denote this function to be $\varphi(I_t)$, i.e.:

$$\begin{aligned} \varphi(I_t) = {} & (1+\rho+\psi\beta)\ln\left(\frac{I_t^i}{1+\rho+\psi\beta}\right) + \beta(1-\psi)\ln\left(\frac{\tilde{e}_{pr}}{\beta\psi}\right) \\ & - (1+\rho+\beta)\ln\left(\frac{I_t^i - \tilde{e}_{pu} + \frac{g+\tilde{e}_{pu}}{\psi}}{1+\rho+\beta}\right) \end{aligned} \tag{37}$$

[11]Proof in Appendix

Formally, there exists two solutions that solves $\varphi(I_t) = 0$ under certain condition as specified in the following proposition:

Proposition 2. *We obtain two solutions for the equation $\varphi(I_t) = 0$, denoted as I_3 and $\bar{I}_3$, when the following condition holds:*

$$(1-\psi)(\tilde{e}_{pr} - \tilde{e}_{pu}) > g \tag{38}$$

Assuming that (38) holds, implies the existence of the threshold I_3 that identifies social classes 3 and 2. The ranking of this threshold with regard to I_4 is:

Proposition 3. *If (38) holds, we get $I_3 < I_4 < \bar{I}_3$.*

Proposition (3) entails a very important corollary: $\bar{I}_3$ is irrelevant as long as it is greater than I_4. This is due to the fact that individuals with income greater than I_4, always choose to belong to social class 4 rather than social class 2 ($V_4 > V_2$).

To determine the optimal social class for each individual, we identify four income thresholds: I_1, I_2, I_3, and I_4. The other income threshold I_0 identifies the subgroup of individuals who belong to social class 1 and do not invest in public education. These thresholds represent the income levels at which individuals are indifferent between belonging to different social classes. Among these, certain thresholds can be systematically ranked. Specifically, we have $I_4 > I_2 > I_0$ and $I_4 > I_3$. However, not all thresholds are directly comparable due to the different constraints inherent in the model (i.e. I_1 and I_3 are not comparable to I_0 and I_2). The thresholds that cannot be sequentially ranked relative to one another give rise to distinct "scenario economies." Each scenario economy represents a unique set of parametric conditions and heterogeneous behaviors, reflecting different configurations of income thresholds. In the following section, I discuss the possible rankings of I_1 and I_3 with respect to I_0 and I_2, and I analyze the consequent implications on the existence and diversity of the aforementioned scenario economies.

It is easy to observe that when I_1 is feasible, I_3 is not, and vice-versa. If $I_1 > I_3$, all individuals with an income falling between these thresholds $I_1 \geq I_t \geq I_3$ would be indifferent between belonging to social classes 2 and 3 which is not possible, as V_2 is equal to V_3 only at I_3. If $I_3 > I_1$, all individuals with an income between I_3 and I_1 would be indifferent between belonging to social class 3 and 1 which is not possible. This implies that I_1 and I_3 do not exist simultaneously in the economy. That is, if we consider one of these two thresholds, we automatically disregard the other. This is also implied for different rankings of I_1 and I_3 with I_0 and I_2.[12]

Consequently, the model has three possible economies. We summarize the possible social group compositions in five economies:

[12]The ranking of I_1 with respect to I_0 and I_2, depends on α thresholds. We define α_1 and α_2 as the

- **Economy 1**: If $I_4 > I_2 > I_0 > I_1$, we get a three-class economy with social classes 1, 3, and 4. The thresholds separating the three groups are I_1 and I_4, respectively. In this economy, all parents in social class 1 invest in public education.

- **Economy 2**: If $I_4 > I_3 > I_2$, we get a four-class economy with social classes 1, 2, 3, and 4. The thresholds separating these groups are I_2, I_3, and I_4 respectively. In this economy, the threshold I_0 identifying social class 1 parents who do not invest in public education is feasible.

- **Economy 3**: If $I_4 > I_1 > I_2$, we get a three-class economy with social classes 1, 2, and 4. The thresholds separating these groups are I_2 and I_4. In this economy, the threshold I_0 is feasible.

- **Economy 4**: If $I_4 > I_2 > I_1 > I_0$, we get a three-class economy with social classes 1, 3, and 4. The thresholds separating these groups are I_1 and I_4. In this economy, the threshold I_0 is feasible.

- **Economy 5**: If $I_4 > I_2 > I_3 > I_0$, we get a three-class economy with social classes 2, 3, and 4. The thresholds separating these groups are I_3 and I_4. In this economy, the threshold I_0 is feasible.

For the remainder of this analysis, we will focus on Economy 1 as our primary scenario because it consists of three groups, making it more representative of the data. Additionally, the dynamics in Economy 1 are applicable to Economy 4. Economy 3 is excluded from the analysis as it lacks social class 3. An analysis of Economy 2, which is applicable to Economy 5, is included in the appendix for reference.

5 Government intervention in education and the middle class

In this section, we focus on the effect of government policies in public education on the middle class for both economies in the short run. Particularly, we center our analysis

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solutions for $I_0 = I_1$ and $I_1 = I_2$, respectively. We get:

$$\alpha_1 = \left(\frac{1+\rho}{1+\rho+\beta\psi}\right)^{\frac{1+\rho+\beta\psi}{\beta}} \quad \text{and} \quad \alpha_2 = \left(1 - \frac{g}{\bar{g}} \cdot \frac{\psi\beta}{1+\rho+\psi\beta}\right)^{\frac{1+\rho+\psi\beta}{\beta}}$$

We can rewrite $\alpha_2 = 1 - \frac{\psi\beta}{(1+\rho+\psi\beta)}$. Since $\frac{g}{\bar{g}} < 1$, we directly get that $\alpha_2 > \alpha_1$. At $g = \bar{g}$ (the case where public education investment in social class 1 is zero) $\alpha_2 = \alpha_1$. When $\alpha < \alpha_1$, we have $I_1 < I_0$. When $\alpha_1 < \alpha < \alpha_2$, we have $I_0 < I_1 < I_2$. When $\alpha > \alpha_2$, we have $I_2 < I_1$. One interesting conclusion is that when public school productivity is low more parents belong to social class 3 (private education), whereas the opposite happens when productivity is high and closer to 1.

around the effect of the marginal change of three parameters on the thresholds I_1, I_3, I_2, and I_4. These parameters are the income tax τ, the per capita government spending on education g, and the quality of public education α. Note that changes in these parameters affect the short-term size of social groups and their respective income. We proceed by analyzing the impact of the parameter changes for each economy separately.

It is important to note that our analysis in this section is concentrated on the short-term effects of these policy changes. We examine how immediate adjustments in policy parameters influence the size and income of children born in the middle class and other social classes without delving into the long-term dynamics. A thorough investigation of the long-term effects would require analyzing the steady-state equilibrium of the economy, which is beyond the scope of this paper.

5.1 Economy 1

In this economy, we have three different groups identified by the threshold I_1 and I_4. Parents' decisions are made based on the level of their income and what social class they belong to. We summarize the three social classes in this economy as follows:

- *Poor:* Has an income below I_1, does not give bequest and invests $e^i_{pu,t}$ in public education (social class 1).
- *Middle class:* Has an income above I_1 but below I_4, does not give bequest and invests $e^i_{pr,t}$ in private education (social class 3).
- *Rich:* Has an income above I_4, gives bequest $b^i_{pr,t+1}$ and invests e^i_{pr} in private education (social class 4).

5.1.1 The marginal effect of government spending in public education g

In this subsection, we analyze how an increase in government spending on public education (g) affects the social class composition in Economy 1, with a particular focus on the middle class. Our objective is to understand the implications of higher public education funding on parents' educational choices and the subsequent impact on their children's human capital formation.

To begin, we examine the effect of an increase in g on the income threshold I_1, which separates the poor class from the middle class. By differentiating I_1 with respect to g, we obtain:

$$\frac{\partial I_1}{\partial g} = \frac{\alpha^{\frac{\beta}{1+\rho+\psi\beta}}}{1-\alpha^{\frac{\beta}{1+\rho+\psi\beta}}} > 0. \tag{39}$$

Since $\alpha \in (0,1)$, both the numerator and the denominator are positive, ensuring that the derivative is positive. This positive relationship indicates that as government spending on public education increases, the threshold I_1 rises. Individuals whose incomes were just above the previous I_1 now find themselves below the new, higher threshold, effectively expanding the size of the poor class. Notably, the threshold I_4, which separates the middle class from the rich class, remains unchanged because it does not depend on g. Therefore, the size of the rich class remains unaffected by changes in government spending on education and the income range defining the middle class narrows, leading to a shrinkage of the middle class.

The magnitude of the threshold shift is influenced by the quality of public education, represented by α. A higher α (closer to 1) amplifies the shift in I_1, resulting in a more significant contraction of the middle class. This outcome is intuitive: as public education becomes more productive, parents perceive greater value in public schooling, leading some to opt out of private education in favor of the improved public option.

Next, we explore the impact of increased government spending on educational investment and human capital levels for the different social classes. For the rich and middle classes, the levels of educational investment remain unchanged. This is evident from equations (18) and (21), which show that the optimal investments $\tilde{e}_{pr}$ and $\hat{e}_{pr}$ are independent of g. As a result, the human capital of the next generation within these classes is unaffected by changes in government spending.

In contrast, the poor class exhibits a different response. Parents in this class adjust their educational investment in reaction to changes in g. Differentiating $\hat{e}_{pu}$ with respect to g, we find:

$$\frac{\partial \hat{e}_{pu}}{\partial g} = -\frac{1+\rho}{1+\rho+\psi\beta} \in (-1, 0). \tag{40}$$

This negative relationship implies that as government spending on education increases, parents in the poor class reduce their own investment $\hat{e}_{pu}$, but not by as much as the increase in g. This partial offset occurs because government spending and parental investment are imperfect substitutes in the human capital production function, as shown in equation (3). Despite reducing their own spending, the total educational resources available to their children $(g + \hat{e}_{pu})$ increase, leading to an improvement in the children's human capital and future income.

The reduction in parental investment allows parents in the poor class to reallocate resources toward their own lifetime consumption (c_t and d_{t+1}). The extent of this reallocation and the trade-off between investing in children's education versus personal consumption depend on two key parameters: the altruism parameter (β) and the concavity parameter of the human capital production function (ψ).

Specifically, when parents are less altruistic (lower β), they place greater emphasis on current consumption over investing in their children's future income. In this case, the reduction in $\hat{e}_{pu}$ approaches the full amount of the increase in g ($\left|\frac{\partial \hat{e}_{pu}}{\partial g}\right| \approx 1$). Conversely, if parents are more altruistic (higher β), they are less inclined to reduce their investment in education, resulting in a smaller decrease in $\hat{e}_{pu}$ ($\left|\frac{\partial \hat{e}_{pu}}{\partial g}\right| \approx 0$).

The concavity parameter ψ also plays a crucial role. A higher ψ indicates that the human capital production function is less concave, meaning that additional spending on education yields higher marginal returns. In such cases, parents are less likely to reduce their educational investment when g increases, as the benefits of additional investment are more pronounced.

An important result is the impact on middle-class individuals whose incomes are just above I_1 before the increase in g. As I_1 rises, some of these individuals find it optimal to switch from private to public education, effectively moving from the middle class to the poor class. This shift results in a decrease in their children's human capital, as they now receive lower-quality education (by the assumption that private education is superior) and benefit less from parental investment. Using (2), (3), (21) and (22), we can easily show that the income of children whose parents belong to social class 3 is higher than the income of children whose parents belong to social class 1.

From the parents' perspective, this change represents a net gain in utility. By opting for public education, they can reduce educational expenses and increase their own consumption.

These results are consistent with the evidence presented in Section 2, where it is shown that middle-class families tend to opt out of private education as government spending on public education increases. The theoretical framework developed here provides a rationalization for this behavior.

5.1.2 The Marginal Effect of Taxation (τ)

The impact of changes in taxation (τ) on different social classes is analyzed through its effect on income thresholds and investment decisions. Notably, the threshold I_1, which separates the poor from the middle class, remains unaffected by changes in taxation since it is independent of τ. Consequently, the size of the poor class remains constant, as does their level of investment in education.

However, the threshold I_4, which separates the middle class from the rich class, is influenced by changes in taxation. Differentiating I_4 with respect to τ yields:

$$\frac{\partial I_4}{\partial \tau} = \frac{-(1+\rho+\psi\beta)}{\psi\beta(1-\psi)(1-\tau)} \left(\frac{(1-\tau)w\psi}{R}\right)^{\frac{1}{1-\psi}} < 0 \tag{41}$$

This result indicates that, as taxes increase, I_4 decreases. The reduction in I_4 decreases the size of the middle class, as some individuals near the upper end of this class transition into the rich class. This shift is primarily driven by changes in the optimal level of private education investment for the rich, $\tilde{e}_{pr}$, which is negatively correlated with taxation:

$$\frac{\partial \tilde{e}_{pr}}{\partial \tau} = \frac{-1}{(1-\psi)(1-\tau)} \left(\frac{(1-\tau)w\psi}{R} \right)^{\frac{1}{1-\psi}} < 0 \tag{42}$$

An increase in taxes lowers the optimal private education investment $\tilde{e}_{pr}$, making it more affordable for a subset of middle-class individuals. These individuals, whose incomes were previously marginally below I_4, now find it optimal to invest at the rich class level. Consequently, they start exhibiting behavior typical of the rich class, such as making the optimal private education investment and providing bequests.

For individuals already classified as rich, an increase in taxation reduces their optimal level of private education investment, leading to a decrease in their children's human capital. To mitigate this effect, rich parents increase the bequests they leave to their children. This adjustment partially offsets the decline in the children's future income caused by the reduced investment in education. However, the compensatory bequest does not fully make up for the loss in human capital, eventually leading to a reduced income for the children. We can illustrate this effect by differentiating the child's income with respect to the tax rate τ:

$$\begin{aligned} \frac{\partial I_{t+1}}{\partial \tau} &= \frac{\partial h_{t+1}}{\partial \tau} + \frac{\partial b_{pr,t+1}}{\partial \tau}, \\ \text{where} \quad \frac{\partial h_{t+1}}{\partial \tau} &= \frac{-1}{(1-\psi)(1-\tau)} \tilde{e}_{pr}, \\ \text{and} \quad \frac{\partial b_{pr,t+1}}{\partial \tau} &= \left| \frac{\partial h_{t+1}}{\partial \tau} \right| \cdot \frac{1+\rho+\psi\beta}{1+\rho+\beta}. \end{aligned} \tag{43}$$

This demonstrates that while higher taxes decrease the children's human capital (since $\frac{\partial h_{t+1}}{\partial \tau} < 0$), parents increase their bequests (as $\frac{\partial b_{pr,t+1}}{\partial \tau} > 0$) to offset this effect. However, because the bequest does not fully compensate for the reduction in human capital, the children's total income still decreases.

5.1.3 The marginal effect of a change in quality α

An increase in the quality of public education (α) has a significant impact on the income threshold I_1 that separates the poor from the middle class. By differentiating I_1 with respect to α, we find:

$$\frac{\partial I_1}{\partial \alpha} = g \frac{\beta}{1+\rho+\psi\beta} \left(\frac{\alpha^{\frac{\beta}{1+\rho+\psi\beta}-1}}{(1-\alpha^{\frac{\beta}{1+\rho+\psi\beta}})^2} \right) \tag{44}$$

This result implies that an increase in the quality of public education leads to a reduction in the size of the middle class. Similar to the effect of increased government spending g, a marginal group within the middle class opts out of private schooling, preferring higher indirect utility over a better lifetime income for their children. From equation (44), we observe that the impact of the change in quality is more pronounced when parents place less importance on the future income of their descendants. That is, when β is smaller.

However, unlike the effect of increased government spending, an improvement in α enhances the productivity of both government and parental investment in education. Consequently, the optimal amount of investment in public education for adults belonging to the poor class, denoted as $\hat{e}_{pu}$, remains unchanged, and the lifetime income of their children increases. We show this by taking the derivative of the future income with respect to τ for social class 2:

$$\frac{\partial I_{t+1}}{\partial \tau} = (1-\tau)w(g+\hat{e}_{pu}) > 0 \tag{45}$$

6 Conclusion

In this paper, we empirically demonstrate that, in the U.S., the middle class's decision regarding the type of education for their children is more sensitive to changes in government spending than that of the poor and rich classes. Specifically, our findings show that increases in public education spending lead middle-class families to opt out of private schools more than other social classes. Motivated by this empirical evidence, we present a model of overlapping generations where parents care about the welfare of their direct descendants and contribute to their income by either investing in their education or giving them a direct transfer in the form bequest. We assume that parents can invest in public or private education depending on their stock of wealth. Moreover, we introduce a parameter α capturing the quality of public education and impose that it is strictly lower than one to establish the superiority of private education. In addition, we assume that individuals in this economy are credit-constrained, so they cannot borrow to invest in education. This implies that if a poor individual cannot afford private education, they would invest in public education or not invest at all.

We deduce that different social stratifications are possible and we focus on one case that we consider is particularly relevant. One with four classes and one with three classes. These classes are differentiated by their levels of human capital and bequest as well as subsequent transfers to their offsprings. We compute the income thresholds defining the different social classes. Parents choose an education system, invest in education and decide to leave bequest or not, depending on whether their endowed income is below or above the threshold computed. We use these thresholds to analyze the effect of three main policies on the distribution of individuals across the different classes. Particualrly, we analyse the impact of government spending on education, taxes, and productivity of the public education.

The model shows that higher public spending can unintentionally lower the future income of middle-class children by encouraging a shift from private to less productive public education. This outcome provides a rationale for the behaviors documented in our data, where such a transition reduces the size of the middle class by limiting opportunities for upward mobility. We also examine how policies like changes in income tax rates and improvements in public education quality affect the different social classes.

These findings highlight the complexity of education policy and its varied impacts on different social groups. Policymakers need to consider these differences to avoid unintended negative effects on the middle class, which is crucial for reducing inequality. Future research should explore ways to improve public education quality without disadvantaging the middle class and investigate the long-term effects of these educational choices on economic inequality and social mobility. This policy analysis could be done in a quantitative model that incorporates public and private education decisions and generates social classes.

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Appendices

Appendix A First order conditions

We derive the optimal conditions on c_t^i, d_{t+1}^i, e_t^i, s_t^i and b_{t+1}^i. We maximize ((1)) with respect to $\{c_t^i, d_{t+1}^i, e_t^i, b_{t+1}^i\}$ subject to (4), (5), (6) and (7). As result we get the following Lagrangian with λ_t as the lagrangian multiplier:

$$\mathcal{L}_t = \ln c_t^i + \rho \ln d_{t+1}^i + \beta \ln I_{t+1}^i + \lambda_t \left((1-\tau) w h_t^i + b_t^i - c_t^i - \frac{d_{t+1}^i + b_{t+1}^i}{R_{t+1}} - e_t^i \right) \tag{A.1}$$

The first-order conditions (FOC) of this problem are given by:

$$c_t^i = \frac{1}{\lambda_t} \tag{A.2}$$

$$d_{t+1}^i = \frac{\rho R}{\lambda_t} \tag{A.3}$$

$$\frac{\beta(1-\tau) w \alpha \psi (\bar{g} + e)^{\psi - 1}}{I_{t+1}^i} = \lambda_t \quad \text{For public education} \tag{A.4}$$

$$\frac{\beta(1-\tau) w \psi e^{\psi - 1}}{I_{t+1}^i} = \lambda_t \quad \text{For private education} \tag{A.5}$$

and

$$I_{t+1}^i \geq \frac{\beta R}{\lambda_t} \tag{A.6}$$

By substituting Equation (A.2) into Equation (6), we obtain the optimal level of c_t as follows:

$$I_t = c_t + \frac{\rho R c_t + b_{t+1}}{R} + e_t \tag{A.7a}$$

$$\Rightarrow c_t = \frac{I_t - \frac{b_{t+1}}{R} - e_t}{1 + \rho} \tag{A.7b}$$

Optimal level of d_{t+1}:

$$d_{t+1} = \rho R c_t. \tag{A.8}$$

We know that when $b_{t+1} > 0$, (A.6) holds at equality. Thus,

$$I_t = \beta R c_t \tag{A.9a}$$

$$\Rightarrow (1-\tau)wh_{t+1} + b_{t+1} = \beta R \left(\frac{I_t - \frac{b_{t+1}}{R} - e_t}{1+\rho} \right) \tag{A.9b}$$

$$\Rightarrow b_{t+1} = \frac{\beta R(I_t - e_t) - (1+rho)(1-\tau)wh_{t+1}}{1+\rho+\beta}. \tag{A.9c}$$

Note that b_{t+1} has to be positive. This implies $\beta R(I_t - e_t) > (1+rho)(1-\tau)wh_{t+1}$.

Now we compute the optimal levels of education for public and private when $b_{t+1} = 0$ and $b_{t+1} > 0$. First, we have:

$$\frac{\partial \mathcal{L}_t}{\partial e_{pu}} = 0 \quad \Rightarrow \quad \frac{\beta(1-\tau)w\alpha\psi(g+e_{pu})^{\psi-1}}{I_{t+1}} = \lambda_t, \tag{A.10}$$

and

$$\frac{\partial \mathcal{L}_t}{\partial e_{pr}} = 0 \quad \Rightarrow \quad \frac{\beta(1-\tau)w\psi(e_{pr})^{\psi-1}}{I_{t+1}} = \lambda_t. \tag{A.11}$$

When $b_{t+1} > 0$ we use (A.6) with equality and substitute in the (A.10) and (A.11). We get:

$$\beta(1-\tau)w\alpha\psi(g+\tilde{e}_{pu})^{\psi-1} = \lambda_t \frac{\beta R}{\lambda_t} \tag{A.12a}$$

$$\beta(1-\tau)w\alpha\psi(g+\tilde{e}_{pu})^{\psi-1} = \beta R \tag{A.12b}$$

$$(g+\tilde{e}_{pu})^{\psi-1} = \frac{(1-\tau)w\alpha\psi}{R} \tag{A.12c}$$

$$g+\tilde{e}_{pu} = \left(\frac{(1-\tau)w\alpha\psi}{R} \right)^{\frac{1}{1-\psi}} \tag{A.12d}$$

$$\tilde{e}_{pu} = \left(\frac{(1-\tau)w\alpha\psi}{R} \right)^{\frac{1}{1-\psi}} - g \tag{A.12e}$$

$$\beta(1-\tau)w\psi(\tilde{e}_{pr})^{\psi-1} = \lambda_t \frac{\beta R}{\lambda_t} \tag{A.13a}$$

$$\beta(1-\tau)w\psi(\tilde{e}_{pr})^{\psi-1} = \beta R \tag{A.13b}$$

$$(\tilde{e}_{pr})^{\psi-1} = \frac{(1-\tau)w\psi}{R} \tag{A.13c}$$

$$\tilde{e}_{pr} = \left(\frac{(1-\tau)w\psi}{R} \right)^{\frac{1}{1-\psi}}. \tag{A.13d}$$

When $b_{t+1} = 0$, we use (A.9b) and substitute in (A.10) and (A.11), we get:

$$\frac{\beta(1-\tau)w\alpha\psi(g+\hat{e}_{pu})^{\psi-1}}{I_{t+1}} = \frac{1}{c_t} \tag{A.14a}$$

$$\frac{\beta(1-\tau)w\alpha\psi(g+\hat{e}_{pu})^{\psi-1}}{(1-\tau)w\alpha(g+\hat{e}_{pu})^{\psi}} = \frac{1+\rho}{I_t - \frac{b_{t+1}}{R} - \hat{e}_{pu}} \tag{A.14b}$$

$$\frac{\beta\psi}{g+\hat{e}_{pu}} = \frac{1+\rho}{I_t - \hat{e}_{pu}} \tag{A.14c}$$

$$\hat{e}_{pu} = \frac{\beta\psi I_t - (1+\rho)g}{1+\rho+\beta} \tag{A.14d}$$

$$\tag{A.14e}$$

$$\frac{\beta(1-\tau)w\psi(\hat{e}_{pr})^{\psi-1}}{I_{t+1}} = \frac{1}{c_t} \tag{A.15a}$$

$$\frac{\beta(1-\tau)w\psi(\hat{e}_{pr})^{\psi-1}}{(1-\tau)w(\hat{e}_{pr})^{\psi}} = \frac{1+\rho}{I_t - \frac{b_{t+1}}{R} - \hat{e}_{pr}} \tag{A.15b}$$

$$\frac{\beta\psi}{\hat{e}_{pr}} = \frac{1+\rho}{I_t - \hat{e}_{pr}} \tag{A.15c}$$

$$\hat{e}_{pr} = \frac{\beta\psi I_t}{1+\rho+\beta} \tag{A.15d}$$

Appendix B Proof for proposition 1

Proof. We know that $V_4(I_t)$ is defined such that $I_t \in]I_4, +\infty[$. Computing the limit when I_t goes to I_4, we get :

$$\lim_{I_t \to I_4} V_4 = (1+\rho+\beta)\ln(\frac{\tilde{e}_{pr}}{\psi\beta}) + \rho\ln\rho R + \beta\ln\beta R$$

we know that $V_3(I_t)$ is defined over $\mathbb{R}^+$. Computing $V_3(I_4)$ we get:

$$V_3(I_4) = (1+\rho+\beta)\ln(\frac{\tilde{e}_{pr}}{\psi\beta}) + \rho\ln\rho R + \beta\ln\beta R$$

Notice that

$$\lim_{I_t \to I_4} V_4 = V_3(I_4)$$

We then compute the slope of both functions with respect of I_t, we get:

$$\frac{\partial V_4}{\partial I_t} = \frac{1+\rho+\beta}{I_t + (\frac{1-\psi}{\psi})\tilde{e}_{pr}} \quad \text{and,} \quad \frac{\partial V_3}{\partial I_t} = \frac{1+\rho+\psi\beta}{I_t}$$

It is easy to show that $\frac{\partial V_4}{\partial I_t} > \frac{\partial V_3}{\partial I_t}$ if and only if $I_t > I_4$. Consequently, $V_4 > V_3$ if $I_t > I_4$ Similarly,We know that $V_2(I_t)$ is defined such that $I_t \in]I_2, +\infty[$. Computing the limit when

I_t goes to I_2, we get :

$$\lim_{I_t \to I_2} V_2 = (1 + \rho + \psi\beta)\ln(\frac{\tilde{e}_{pu} + g}{\psi\beta}) + \rho \ln \rho R + \beta \ln \beta R$$

we know that $V_1(I_t)$ is defined over $\mathbb{R}^+$. Computing $V_1(I_2)$ we get:

$$V_1(I_2) = (1 + \rho + \psi\beta)\ln(\frac{\tilde{e}_{pu} + g}{\psi\beta}) + \rho \ln \rho R + \beta \ln \beta R$$

Notice that

$$\lim_{I_t \to I_2} V_2 = V_1(I_2)$$

We then compute the slope of both functions with respect of I_t, we get:

$$\frac{\partial V_2}{\partial I_t} = \frac{1 + \rho + \beta}{I_t - \tilde{e}_{pu} + \frac{g + \tilde{e}_{pu}}{\psi}} \quad \text{and,} \quad \frac{\partial V_1}{\partial I_t} = \frac{1 + \rho + \psi\beta}{I_t + g}$$

It is easy to show that $\frac{\partial V_2}{\partial I_t} > \frac{\partial V_1}{\partial I_t}$ if and only if $I_t > I_2$. Consequently, $V_2 > V_1$ if $I_t > I_2$

All in all,

$$\begin{cases} I_t > I_4 \Leftrightarrow V_4 > V_3 \\ I_t > I_2 \Leftrightarrow V_2 > V_1 \end{cases} \tag{B.1}$$

□

Appendix C Proof for proposition 2

Proof. Using (37) we compute the derivative with respect of I_t. We get :

$$\frac{\partial \varphi}{\partial I_t} = \frac{(1 + \rho + \psi\beta)(I_t - \tilde{e}_{pu} + \frac{g + \tilde{e}_{pu}}{\psi}) - (1 + \rho + \beta)I_t}{I_t(I_t - \tilde{e}_{pu} + \frac{g + \tilde{e}_{pu}}{\psi})}$$

We then solve $\frac{\partial \varphi}{\partial I_t} = 0$ to obtain the income at which the function φ reaches its maximum level. We call this income level I_m:

$$I_m = \frac{(1 + \rho + \psi\beta)((1 - \psi)\tilde{e}_{pu} + g)}{(1 - \psi)\psi\beta}$$

As such, when $I_t < I_m$, $\varphi(I_t)$ is increasing, and when $I_t > I_m$, $\varphi(I_t)$ is decreasing. We can conclude with ease that the function $\varphi(I_t)$ will intersect twice with the horizontal axis if and only if there exists a range of I_t for which $\varphi(I_t)$ is strictly positive.

We compute $\varphi(I_m)$, and we get:

$$\varphi(I_m) = (1 - \psi)\beta \ln \left(\frac{(1 - \psi)\tilde{e}_{pr}}{(1 - \psi)\tilde{e}_{pu} + g} \right)$$

If $\varphi(I_m) > 0$ there exists two points at which $\varphi(I_t)$ intersect with the horizontal axis, let us denote them I_3 and $\bar{I}_3$ such that $I_3 < \bar{I}_3$. This implies the following condition:

$$(1-\psi)(\tilde{e}_{pr} - \tilde{e}_{pu}) > g$$

If this condition is held with equality $\varphi(I_t)$ intersect with the horizontal axis in one point which is I_m. and if this condition is reversed $\varphi(I_t)$ does not intersect at all with with the horizontal axis. □

C.1 Proof for proposition 3

Proof. We know that the maximum of $\varphi(I_t)$ is:

$$I_m = \frac{(1+\rho+\psi\beta)((1-\psi)\tilde{e}_{pu} + g)}{(1-\psi)\psi\beta}$$

such that $I_3 < I_m < \bar{I}_3$ when (38) holds. Comparing I_4 from (29) with I_m, we easily get that:

$$I_4 > I_m \Leftrightarrow (1-\psi)(\tilde{e}_{pr} - \tilde{e}_{pu}) > g$$

Consequently,

$$I_3 < I_m < I_4.$$

Substituting I_4 in $\varphi(I_t)$, we get that $\varphi(I_4) > 0$ if and only if $(1-\psi)(\tilde{e}_{pr} - \tilde{e}_{pu}) > g$. And since we know that for $I_t > \bar{I}_3$ we have $\varphi(I_t) < 0$, we deduce that $I_4 < \bar{I}_3$ □

Appendix D Economy 2

In this economy, we have four different groups identified by the threshold I_2, I_3, and I_4. Parents' decisions are made based on the level of their income and the social class they belong to. We summarize the four social in this economy as follows:

- *Poor:* has income below I_2, does not give bequest and invests $e^i_{pu,t}$ in public education.
- *Lower middle class:* has income above I_2 and below I_3, gives bequest $b^i_{pu,t+1}$ and invests e^i_{pu} in public education.
- *Upper middle class:* has income above I_3 and below I_4, does not give bequest and invests $e^i_{pr,t}$ in private education.
- *Rich:* has income above I_4, gives bequest $b^i_{pr,t+1}$ and invests e^i_{pr} in private education.

D.1 The marginal effect of government spending in public education g

Differentiating I_2 and I_3, we get:

$$\frac{\partial I_2}{\partial g} = -1 < 0 \tag{D.1}$$

and,

$$\frac{dI_3}{dg} = -\frac{\frac{\partial \varphi}{\partial g}}{\frac{\partial \varphi}{\partial I}} = \frac{(1+\rho+\beta)I}{(1+\rho+\psi\beta)(I-\tilde{e}_{pu}+\frac{g+\tilde{e}_{pu}}{\psi})-(1+\rho+\beta)I} > 0 \tag{D.2}$$

(D.1) and (D.2) imply that the size of the poor class and the upper middle class reduces and the size of the lower middle class increases as g increases. In contrast, the size of the rich class remains unchanged since I_4 does not depend on government expenditure. The change in policy also impacts the optimal level of investment in public education for both the poor and the lower middle class. This partially explains why the size of the first social class adjusts simultaneously. As I_2 decreases, a marginal group from the poor class finds it optimal to switch to the lower middle class. Similarly, the decrease in the optimal investment in public education $\tilde{e}_{pu}$ instigates a group in the upper middle class to switch to the lower middle class as investing in public education and giving bequest gives more utility now. Note that an increase in government expenditure increases the optimal level of consumption for the poor and the lower middle class. In terms of the income of the next generations, we take into consideration what happens to bequest. Since this latter does not change for both the lower middle class and the rich, I can conclude that the future income of the lower middle class and the future income of the rich remain unchanged. The future income of the lower middle class is not affected because education and government spending are perfect substitutes, whereas the future income of the rich is unaffected simply because spending on private education and bequests given by this social class do not depend on government spending.

D.2 The marginal effect of taxation τ

$$\frac{\partial \hat{e}_{pu}}{\partial \tau} = \frac{-1}{(1-\psi)(1-\tau)}\left(\frac{(1-\tau)w\alpha\psi}{R}\right)^{\frac{1}{1-\psi}} \tag{D.3}$$

$$\frac{\partial \hat{e}_{pr}}{\partial \tau} = -\frac{w\psi}{R(1-\psi)}\hat{e}_{pr}^{\psi} \tag{D.4}$$

$$\frac{\partial I_2}{\partial \tau} = -\frac{(1+\rho+\psi\beta)w\alpha}{\beta(1-\psi)}(\hat{e}_{pu}+\bar{g})^{\psi} \tag{D.5}$$

$$\frac{dI_3}{d\tau} = -\frac{\frac{\partial \varphi}{\partial \tau}}{\frac{\partial \varphi}{\partial I_3}} = -\left(\frac{\frac{I_3}{\psi(1-\tau)}[(1+\rho+\beta)\tilde{e}_{pu}-\beta\psi(I_3-\tilde{e}_{pu}+\frac{\bar{g}+\tilde{e}_{pu}}{\psi})]}{(1+\rho+\psi\beta)(I_3-\tilde{e}_{pu}+\frac{\bar{g}+\tilde{e}_{pu}}{\psi})-(1+\rho+\beta)I_3}\right) \tag{D.6}$$

D.3 The marginal effect of the quality of public education α

$$\frac{\partial I_2}{\partial \alpha} = \frac{1+\rho+\psi\beta}{\psi\beta\alpha(1-\psi)}\left(\tilde{e}_{pu}+g\right) \tag{D.7}$$

$$\frac{dI_3}{d\alpha} = -\frac{\frac{\partial \varphi}{\partial \tau}}{\frac{\partial \varphi}{\partial I_3}} = \frac{\frac{I_3}{\psi\alpha}(1+\rho+\beta)\tilde{e}_{pu}}{(1+\rho+\psi\beta)(I_3-\tilde{e}_{pu}+\frac{g+\tilde{e}_{pu}}{\psi})-(1+\rho+\beta)I_3} \tag{D.8}$$

$$\frac{\partial \hat{e}_{pu}}{\partial \alpha} = \frac{1}{(1-\psi)\alpha}\left(\frac{(1-\tau)w\alpha\psi}{R}\right)^{\frac{1}{1-\psi}} \tag{D.9}$$

D.4 Data

- All dollar measures are at constant dollars year 2019 using the R-CPI-U-RS produced by the U.S. Bureau of Labor Statistics.
- Data on public elementary-secondary education finance for the years 2005-2019 is collected from the Annual Survey of School System Finances.
- Household measures are collected from American Community Surveys (ACS) from 2005 to 2019.

Appendix E Robustness

Table 2: Choice of Private Schooling on Total public spending per capita

	Middle class (1)	Rich (2)	Poor (3)	Middle class (4)	Rich (5)	Poor (6)
Total Pub spending per capita	-0.385***	-0.126	-0.264	-0.353***	-0.094	-0.250
	(-4.798)	(-1.333)	(-1.540)	(-3.948)	(-1.024)	(-1.389)
Household Income	0.690***	0.860***	0.008	0.406***	0.783***	-0.010
	(14.782)	(29.417)	(0.636)	(13.766)	(28.000)	(-1.479)
Household educ	-	-	-	0.679***	0.621***	0.759***
				(23.582)	(15.135)	(19.952)
Metropolitan dummy	0.283***	0.405***	0.176***	0.326***	0.385***	0.335***
	(7.871)	(8.920)	(5.395)	(9.309)	(8.615)	(7.464)
Observations	2633037	1318289	1287456	2633037	1318289	1287456
Time fixed effects	✓	✓	✓	✓	✓	✓
State fixed effects	✓	✓	✓	✓	✓	✓
Race controls	✗	✗	✗	✓	✓	✓

Notes: The dependent variable is a dummy that takes 1 if the household has at least one child in private school. The total public expenditure per capita and household income are expressed in logs of constant 2019 dollars. Covariates include household income and race dummies as well as time and age-fixed effects. The poor and rich groups represent the first and last quartiles, respectively. The middle class represents the second and third quartiles. For data sources and summary statistics, see Appendix. Standard errors are clustered at the state level and are reported in brackets. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, *** indicates significance at the 1 percent level.